

Analysis of discrepancies in Dalitz plot parameters in $\eta \rightarrow 3\pi$ decay[☆]

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Abstract

We analyze the Dalitz plot parameters of $\eta \rightarrow 3\pi$ decay in the framework of resummed chiral perturbation theory. This approach allows us to keep the uncertainties in the NNLO and higher orders under better control and estimate their influence. We cannot confirm the suspected discrepancy in the case of the charged decay parameter b , where even small uncertainties in higher orders could accommodate the difference. On the other hand, we find the experimental value of the neutral decay parameter α incompatible with an assumption of good convergence properties in the center of the Dalitz plot. We calculate $\pi\pi$ rescattering bubble corrections up to three loops and show that these might explain the discrepancy, especially for a low value of the pseudoscalar decay constant in the chiral limit. However, that could indicate a failure of convergence of the chiral series in this channel already at low energies around 500MeV.

Keywords:

chiral perturbation theory, eta meson, $\pi\pi$ rescattering

Chiral perturbation theory [1, 2] has a long history [3] of trying to explain experimental data on the $\eta \rightarrow 3\pi$ decay. As is clear from the decay rate calculations, the theory converges really slowly for this decay channel. While the latest NNLO calculations [4] provide reasonable predictions for this quantity, experimental data are being gathered with increasing precision in order to make more detailed analysis of the Dalitz plot distribution possible. The conventional Dalitz plot parameters are defined as (we generally follow notations from [4]):

$$\eta \rightarrow \pi^0 \pi^+ \pi^- : |A|^2 = A_0^2(1 + ay + by^2 + dx^2 + \dots) \quad (1)$$

$$\eta \rightarrow 3\pi^0 : |\bar{A}|^2 = \bar{A}_0^2(1 + \alpha z + \dots) \quad (2)$$

where $x \sim u-t$, $y \sim s_0-s$, $z \sim x^2+y^2$ and s_0 is the Dalitz plot center $s_0 = 1/3(M_\eta^2 + 3M_\pi^2)$.

Comparison of the recent experimental information with the NNLO χ PT results can be found in tables 1, 2. As can be seen, the most obvious discrepancy appears in the charged decay parameter b and the neutral decay parameter α .

Other approaches were developed in order to model the amplitude better, namely dispersive approaches [14,

		α
1998	Crystal Barrel [7]	-0.052 ± 0.020
2001	Crystal Ball [8]	-0.031 ± 0.004
2007	WASA at CELSIUS [9]	-0.026 ± 0.014
2009	WASA at COSY [10]	-0.027 ± 0.009
2009	Cr.Ball at MAMI-B [11]	-0.032 ± 0.003
2009	Cr.Ball at MAMI-C [12]	-0.0322 ± 0.0025
2010	KLOE [13]	-0.0301 ± 0.0050
2007	NNLO χ PT [4]	$+0.013 \pm 0.032$

Table 1: Recent experimental and χ PT results for the neutral channel.

	1998	2008	2008
	Cr.Barrel [5]	KLOE [6]	NNLO χ PT [4]
a	-1.22 ± 0.07	-1.090 ± 0.020	-1.271 ± 0.075
b	0.22 ± 0.11	0.124 ± 0.012	0.394 ± 0.102
d	0.06 ± 0.04	0.057 ± 0.017	0.055 ± 0.057

Table 2: Recent experimental and χ PT results for the charged channel.

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15, 16, 17] and non-relativistic effective field theory [18, 19, 20]. These more or less abandon strict equivalence to χ PT and their success in reproducing a negative sign for α can serve as a motivation to ask what is the culprit of the failure of chiral perturbation theory to do so.

Our aim is thus not to produce another alternative method, but to try to understand whether the theory, by which we mean χ PT as a low energy representation of QCD, really does have difficulties explaining the data and if so, try to identify the source of the problem.

The starting point is the realization that the standard approach to χ PT, as a usual treatment of perturbation series, implicitly assumes good convergence properties and hides the uncertainties associated with a possible violation of this assumption. Error bars are often not reported and it should be stressed that even the ones cited in the NNLO χ PT result above [4] are not systematic uncertainties inherent in the theory, but rather fitting errors tied to numerical procedure peculiar to the method used by the authors.

There is a long standing suspicion that chiral perturbation theory might possess a slow or irregular convergence in the case of the three quark flavor series [21, 22], the $\eta \rightarrow 3\pi$ decay rate might serve as a prime example. An alternative approach, dubbed resummed χ PT [23, 24], was developed in order to express these assumptions in terms of parameters and uncertainty bands. The procedure can be very shortly summarized in the following way:

- standard χ PT Lagrangian and power counting
- only expansions derived directly from the generating functional trusted
- explicitly to NLO, higher orders collected in remainders
- remainders retained, treated as sources of error
- manipulations in non-perturbative algebraic way

Our calculation closely follows the procedure outlined in [25]. What we present here is only a brief excerpt, skipping all the details and concentrating only on the cases of the Dalitz plot parameters b and α . A more comprehensive work is in preparation [32].

Within the formalism, we start by expressing the charged decay amplitude in terms of the 4-point Green functions G_{ijkl} . We compute at first order in isospin breaking, in this case the amplitude takes the form

$$F_\pi^3 F_\eta A(s, t, u) = G_{+-83} - \varepsilon_\pi G_{+-33} + \varepsilon_\eta G_{+-88} + \Delta_{G_D}^{(6)}, \quad (3)$$

where $\Delta_{G_D}^{(6)}$ is the direct higher order remainder to the complete 4-point Green function. The physical mixing

angles to all chiral orders and first in isospin breaking can be expressed in terms of quadratic mixing terms of the generating functional to NLO and related indirect remainders

$$\varepsilon_{\pi,\eta} = -\frac{F_0^2}{F_{\pi^0,\eta}^2} \frac{(\mathcal{M}_{38}^{(4)} + \Delta_{M_{38}}^{(6)}) - M_{\eta,\pi^0}^2 (Z_{38}^{(4)} + \Delta_{Z_{38}}^{(6)})}{M_\eta^2 - M_{\pi^0}^2}. \quad (4)$$

In this approximation the neutral decay channel amplitude can be related to the charged one as

$$\bar{A}(s, t, u) = A(s, t, u) + A(t, u, s) + A(u, s, t). \quad (5)$$

In accord with the method, strictly $O(p^2)$ parameters appear inside loops, while physical quantities in outer legs. Due to the leading order masses in loops such a strictly derived amplitude has an incorrect analytical structure, cuts and poles being in unphysical places. To account for this, the amplitude is carefully modified using a NLO dispersive representation, the procedure is described in detail in [25].

The next step is the treatment of the low energy constants (LEC's). The leading order ones are expressed in terms of convenient parameters

$$Z = \frac{F_0^2}{F_\pi^2}, \quad X = \frac{2F_0^2 B_0 \hat{m}}{F_\pi^2 M_\pi^2}, \quad r = \frac{m_s}{\hat{m}}, \quad R = \frac{(m_s - \hat{m})}{(m_d - m_u)}, \quad (6)$$

where $\hat{m} = (m_u + m_d)/2$. The standard approach tacitly assumes values of X and Z close to one and $r \sim 25$, which means that the leading order terms should dominate the expansion. However, even the most recent standard χ PT fit [26] indicates a much lower values of X and Z , thereby allowing for a possibility of a non-standard scenario of spontaneous chiral symmetry breaking (SB χ S).

At next-to-leading order, the LEC's L_4 - L_8 are algebraically reparametrized in terms of pseudoscalar masses, decay constants and the free parameters X , Z and r using chiral expansions of two point Green functions, similarly to [23]. Because expansions are formally not truncated, each generates an unknown higher order remainder.

We don't have a similar procedure ready for L_1 - L_3 at this point, therefore we collect a set of standard χ PT fits [26, 27, 28] and by taking their mean and spread, while ignoring the much smaller reported error bars, we obtain an estimate of their influence. As will be shown in [32], the results depend on these constants only very weakly. The error bands given here include the estimated uncertainties in L_1 - L_3 .

The $O(p^6)$ and higher order LEC's, notorious for their abundance, are collected in a relatively smaller number of higher order remainders. We also fix the s -quark

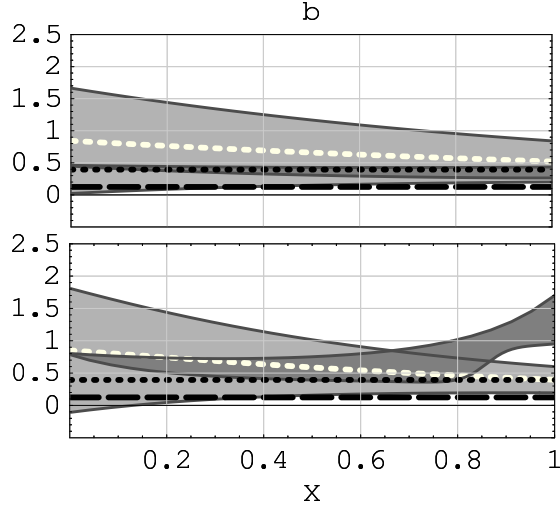


Figure 1: Charged decay parameter b : top $Z=1$, bottom $Z=0.5$
horizontal lines: dashed - KLOE measurement [6]; dotted - $S\chi PT$ [4]
light band - statistical remainder estimate; dark band - result including $\pi\pi$ rescattering, depending on scale $\mu=0.5\div 1\text{GeV}$.

mass at $r=25$, motivated by lattice [29], as its value is anyway connected with X through the Kaplan-Manohar ambiguity [30]. We also investigated a low value $r=15$, but the results does not radically alter the presented conclusions [32]. At last, because at first order in isospin breaking the Dalitz plot parameters do not depend on R , we are left, besides the remainders, with two free parameters: X and Z . These effectively control the scenario of $SB\chi S$ in our results.

Finally, the last step leading to numerical results is the estimate of the remainders. We have a direct remainder to the 4-point Green function and eight indirect ones - three related to both pseudoscalar masses and decay constants, two to mixing angles. We use two approaches. The first one is based on general arguments about the convergence of the chiral series [23], which leads to

$$\Delta_G^{(6)} \sim \pm 0.1G, \quad (7)$$

where G here stands for any of our 2-point or 4-point Green functions, which generate the remainders. This is in principle an assumption. Hence we test the compatibility of this assumption of a reasonably good chiral convergence of trusted quantities with experimental data in a statistical sense. One peculiarity should be noted though - Dalitz plot parameters are derivative quantities in terms of the Mandelstam variables and even if the direct remainder had a small absolute value around the center of the Dalitz plot, it could generate a large correction in derivatives. When making an expansion of the direct remainder around the Dalitz plot

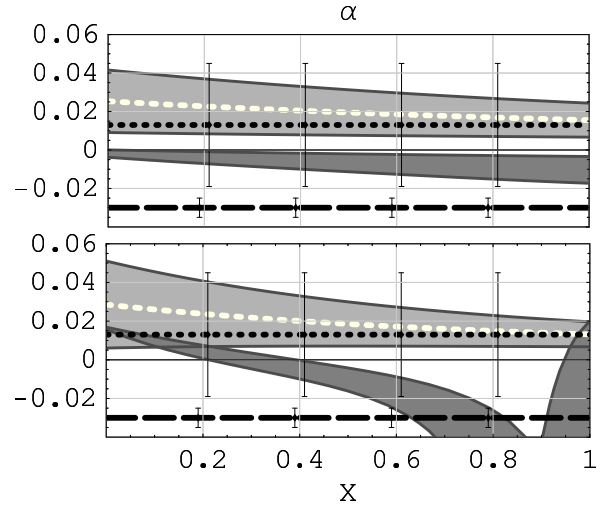


Figure 2: Neutral decay parameter α : top $Z=1$, bottom $Z=0.5$
horizontal lines: dashed - KLOE measurement [6]; dotted - $S\chi PT$ [4]
light band - statistical remainder estimate; dark band - result including $\pi\pi$ rescattering, depending on scale $\mu=0.5\div 1\text{GeV}$.

center, such a large term would either grow to generate a large direct remainder in some other region, possibly unphysical, but still where χPT is assumed to converge well, or could be canceled by terms proportional to higher order derivatives, thus inducing some kind of fluctuation in the amplitude. As we feel such a behavior warrants explanation, we include the absence of unusually large corrections in derivatives in our definition of good convergence properties and test for such a possibility as well.

The results for the statistical remainder estimate are depicted in fig.1 and 2. Several conclusion can be made - in both cases the NNLO standard χPT result lies in our uncertainty bands (the lighter ones), which is an important consistency check. For the charged decay parameter b , this is also true for the latest experimental measurement. This means we cannot confirm any discrepancy, as even a small correction compatible with the assumption reasonable chiral convergence could explain the differences between theory and experiment. On the other hand, the situation is quite different in the case of the neutral decay parameter α , where we can conclude, in accord with the NNLO $S\chi PT$ result, that our definition of good chiral convergence is not compatible with the data. It is also notable that this is true for any value of the free parameters X and Z and hence an alternative scenario of $SB\chi B$, e.g. a small value of the quark condensate, is not the culprit here.

The framework of resummed χPT is well suited to include additional information about higher orders from

various sources [25, 31]. An independent estimate of the remainders can on one side be an important check of the validity of the statistical remainder estimate, as in the case of the parameter b , or could try to explain any deviances from this assumption, which is our aim for α .

While other estimates being in preparation [32], here we employ a specific higher order calculation, namely n -loop bubble contributions to final state $\pi\pi$ rescattering, which generally take the form

$$G_{\pi\pi}^{(2n)} = \sum_{k+l=1}^{n-1} P_{kl}^{(n-k)}(s, t, u) \mu_{\pi}^k J_{\pi\pi}^r(s)^l + (t, u \text{ channels}), \quad (8)$$

where μ_{π} and $J_{\pi\pi}^r(s)$ are the usual chiral logs and one loop scalar functions [2], respectively. $P_{kl}^{(m)}(s, t, u)$ is an m -th order polynomial in the Mandelstam variables. We compute up to $O(p^8)$, that means 3-loop diagrams with LO vertices and 2-loop ones with one or two NLO counter terms. We stress that what we do is not a unitarization procedure but a genuine χ PT calculation. The motivation is that this is one of the suspects that could explain the discrepancy in α [16, 20, 17]. Terms with highest power in s are of the form $\sim N \frac{s^n}{F_0^{2n}} J(s, m_{\pi})^{n-1}$, where N is a numerical factor. In our case we obtain $N^{(1\text{-loop})}=1/2$, $N^{(2\text{-loop})}=4/3$, $N^{(3\text{-loop})}=5/8$, which implies that N is not a suppression factor with LO vertices. Thus at $\left|\frac{s}{F_0^2} J(s, m_{\pi})\right| \approx 1$ convergence blows up, which a simple analysis can show is around $\sqrt{s} \approx 600\text{--}700\text{MeV}$ at $Z=1$ or $\sqrt{s} \approx 400\text{--}500\text{MeV}$ at $Z=0.5$, where the renormalization scale runs through $\mu=0.5\div 1\text{GeV}$.

The result can be seen as the dark bands in figures 1 and 2. If they did not wander outside the light ones, it would indicate a confirmation that the $\pi\pi$ rescattering bubble contributions agree with the statistical remainder estimate. This is indeed true for b , except a small area of the parameter space. Once again the case of α is different and we can see that $\pi\pi$ rescattering could generate a negative sign, especially in a case of small value of the pseudoscalar decay constant in the chiral limit.

Of course, this is very far from a complete calculation to $O(p^8)$, which also expresses itself in dependence on the renormalization scale μ . But the fact that the scale dependence is quite benign could be also interpreted in the way that it is not unreasonable to consider the bubble $\pi\pi$ rescattering separately.

A further discussion can be made [32] and we will only summarize the results briefly - the source of the large negative $\pi\pi$ -rescattering contributions to α turns out to be where suspected, the terms leading in s . These generate a concavity in the amplitude, which is measured precisely by α through the second derivative.

It is interesting to note that the $O(p^8)$ contributions to α are actually larger than the $O(p^6)$ ones, and a quick check of the anticipated form of the s -leading terms in even higher order contributions show that the next few orders can be expected to be large and possibly negative as well. These terms do not induce a large correction to the direct remainder in the center of the Dalitz plot, but the concavity is connected with a quick failure of the convergence of chiral series in the suspected energy region. That is unphysical for the case of the $\eta \rightarrow 3\pi$ decay, but could be an indication, if some other contribution do not cancel them, of a breakdown of the chiral expansion at quite low energies. This could be due to some higher energy structure present, for example a resonance.

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